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Mathematical Programming: A Brief Historical Sketch and its Applications

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Abstract
During the last five decades, Mathematical programming techniques are applied successfully to almost every spare of human activity, such as industries, management, planning, govt and economics, agriculture, engineering, medical science and scientific research. As per Prof. C. R. Rao “All Statistical procedures are, in ultimate analysis, solutions to suitably formulated optimization problems”. In this paper, we give the brief historical sketch of mathematical programming techniques and its applications towards the development of the theory and algorithms for solving various types of mathematical programming problems.

Keywords: Mathematical Programming, Statistical Procedures, Optimization

Optimization

Investigating for and arriving at the best possible decision in any given circumstance is called optimization. The ultimate aim of all such decisions is to maximize the gain or profit, or minimize the cost or loss incurred in certain process. The first step towards optimization is to express the desired benefits, the required efforts and other relevant information as a function of certain variables that may be called "decision variables". Thus optimization can be defined as the maximization or minimization of a function of several variables. This function may be unconstrained or it may be subjected to certain constraints on the variables in the form of equations or inequalities.

The existence of optimization problem can be traced back to the middle of eighteenth century. The work of Newton, Lagrange and Cauchy in solving certain types of optimization problems arising in geometry and physics by using differential calculus methods and calculus of variations is pioneering. These optimization methods better known as classical optimization methods have their own limitations and cannot be applied successfully to every optimization problem. These techniques are mainly of theoretical interest. However, in some simple situations they can provide solutions, which are practically acceptable.

The class of real life optimization problems that are usually not solvable by classical optimization methods are known as mathematical programming problems. In the last five decades there has been a phenomenal advancement towards the development of the theory and algorithms for solving various types of mathematical programming problems.

Mathematical Programming: A Brief Historical Sketch

The first problem in Transportation or Linear Programming was developed by Kantorovich. L.V.[46], a Russian Mathematician and an American economist Hitchcock F. L.[41]. They dealt with a well known transportation problem which forms a branch of linear programming. Even though the French Mathematician Jean. Baptist – Joseph Fourier seemed to be aware of the subject potential as early as 1823. Kantorovich, L.V.[45], published an extensive monograph on mathematical methods in the organization and planning of production in 1939 and is credited with being the first to recognize that certain important broad classes of scheduling problems had well defined mathematical structures. An English economist, Stigler, G.[79], described yet another linear programming problem that of determining an optimal diet not only at the minimum cost but also to satisfy minimum requirements.

The first ever mathematical programming problem (MPP) was perhaps the problem of optimal allocation of limited resources recognized by economists in early 1930s. After World War II in 1947 the United States air force team SCOOP (scientific computation of optimum programs) started intensive research on some optimum resource allocation problem which led to the development of the famous simplex method by George B. Dantzig for solving a linear programming problem (LPP).

In most of the practical situations the integer values of the decision variables are required. Dantzig et al. [20], Markowitz and Manne [61], Dantzig ([22],[23]), etc. discussed the integer solutions to some special purpose LPPs. Gomory ([35],[36]) developed the Cutting plane methods, for whole and mixed integer programming problems. Land and Doig [56] developed the powerful branch and bound technique for solving integer linear programming problems. Later Dakin [19] proposed another interesting variation of Land and Doig algorithm. Hillier [39] gave a bound and scan algorithm, Bowman and Nemhausen [11] gave a modified cutting Plane method, Austin and Grand [5] developed an advanced dual algorithm and Saltzman and Hillier ([74], [75]) presented the exact ceiling point algorithm for solving integer programs. Achuthan and Hill [1] presented eight new cutting planes which provide an improved description of the solution space and they demonstrated the usefulness of these cuts by generating good lower bounds for 14 large literature benchmark problems.

Development of new techniques for solving linear programming problems are still going on. Decades of work on Dantzig's simplex method had failed to yield a polynomial-time variant. The first polynomial-time linear programming algorithm called Ellipsoid Algorithm, developed by Khachiyan [49], opened up the possibility that non-combinatorial methods might beat combinatorial one for linear programming. A new polynomial-time algorithm which generated much excitement in the mathematical community was developed by Karmakar [47]. (An algorithm for which the running time on computer is always less than aL^b, where L is the number of bits required to represent the data in a computer and b some positive number, is called a polynomial time algorithm. On the other hand, if the running time is at the most $C2^L$ for some positive C, we say that it is an exponential time algorithm).

It is claimed that Karmakar's algorithm often outperforms simplex method by a factor of 50 on real world problems. Some recent polynomial time algorithms developed by Renegar [70], Gonzaga [37], Monteiro and Adler [64], Vaidya [81], Reha [69] are faster than Karmakar's algorithm.

Kuhn and Tucker (1951) developed the necessary conditions (which became sufficient also under special circumstances) to be satisfied by an optimal solution of a non-linear programming problem (NLPP). These conditions, known as K-T conditions, laid the foundation for great deal of later research and development in nonlinear programming techniques. Till date no single technique is available which can provide an optimal solution to every NLPP like simplex method for LPP. However different methods are available for some special types of NLPPs. Beale [7] gave a method for solving convex quadratic programming problem (CQPP). One of the power full techniques for solving a NLPP is to transform it by some means, into a form, which permits the use of simplex method of LPP. Using K-T conditions Wolfe [83] transformed the convex quadratic programming problem into an equivalent LPP to which simplex method could be applied with some additional restriction on the vectors entering the basis at various iterations. Some other techniques for solving quadratic programming problems also exist in the literature.

Among other NLPP methods there are gradient methods and gradient projection methods. Like simplex method of LPP these are iterative procedures in which at each step we move from one feasible solution to another in such a way that the value of the objective function is improved. Rosen ([71], [72]), Kelley [48], Goldfarb [34], Du and Zhang [31], Lai et al. [55] etc. gave gradient projection methods for non linear programming with linear and non linear constraints.

Geometric programming (GP), a systematic method for solving the class of mathematical programming problems that tends to appear mainly in engineering design, was first developed by Duffin and Zener in the early 1960s and further extended by Duffin et al. [30]. Davis and Rudolph [25] use GP to optimal allocation of integrated samples in quality control. Also Devis and Finch [25] applied GP to the optimal allocation of Stratified samples with several variance constraints arising from several estimates of deficiency rates in the quality control of administrative decisions.

Goal programming, a well known technique for solving multi objective programming problems was developed by Charnes and Cooper [18]. The short comings and the merits of the solution of the goal programming were discussed by Ignizio [42], Khorramshahgol and Hooshiari [51], Ignizio and Tom [43], Chakraborty and Sinha [13], Neelam and Arora [66], Chakraborty and Dubey [12]. The recent developments in algorithms for solving multi objective linear and non-linear programming problems are due to Sherali [78], Roy and Wallenius [73], Arbel ([2],[3]), Bit et al. [10], Okada [68], Thoudam and Adil [80], etc. Gupta and Chakraborty [38], use the fuzzy programming approach to multi objective linear programming problem.

Several NLP problems consist of the functions, which are separable in nature. A function that can be decomposed additively in terms of single variable function is called a separable function. The methods for the approximate solution to the separable programming problem are found in the works of Charnes and Cooper [15], Markowitz and Manne [61], Dantzig et al. [21], Miller [63], Fleury [33], Megiddo and Tamir [62] etc. The technique applies to problems in which all the non-linear functions are separable. The idea is to construct a constrained optimization model that linearly approximates the original problem. The approximations enlarge the size of the model, but since a version of the simplex method can be applied as a solution technique, the method has considerable practical significance. The approach can be used equally well to approximate a non-linear objective function and non linear constraints.

The principle of optimality enunciated by Bellman [8] paved the way for the development of dynamic programming technique, which has been applied successfully for solving certain special types of MPPs. The problems in which decisions are to be made sequentially at different stages of solutions are called multi stage decision problems. Many multistage decision problems can be formulated as MPP. The dynamic programming technique is a computational procedure, which is well suited for solving MPPs. that may be treated as a multistage decision problem. The main hurdle in using dynamic programming technique to MPPs arising in practical situation is the " problem of Dimensionality." The computational efforts involved increase incredibly fast with the increase in the number of state parameters. However, in some problems of specified nature the problem of dimensionality could be handled efficiently. The dynamic programming became rich and more applicable by the contributions of several authors like Bellman and Dreyfus [9], Wachs [82], Li, D. [57], Li, D. and Haimes [58], Lin, [59], Odanaka [67], Irvine and Sniedovich [44].

Linear fractional programming problems were introduced by Charnels, A., Cooper, W. W. [16]. These types of problems consist of linear type fractional objective function and linear constraint s. Other authors who contributed to this field were Dorn, W. S. [28], Schnabel, S. [76] and Schiabel,S. [77].

For a long time it was not known whether or not, linear programs belonged to a non- polynomial class called ‘hard’ (such as the one, the travelling salesman belongs to) or to a ‘easy’ polynomial class (like the one that the shortest path problem belongs to). Klee V. and Minty G. J. [52] created an example that showed that the classical simplex algorithm would require an exponential number of steps to solve a worst case linear programs by the Russian Mathematician Khanchiang, L. G.[49], developed a polynomial time algorithm for solving linear programs. It is an interior method using ellipsoid inscribed in the feasible region. He proved that the computing time is guaranteed to be less that a polynomial expression in the dimensions of the problem and the number of digits of input data. Although polynomial, the bound he established turned out to be too high for his algorithm to be used to solve practical problems.

Linear programming can be applied to various fields of study. Most extensively, it is used in business and economic situation and engineering problems. Some industries that use linear programming models include transportation, energy, telecommunication, production or manufacturing companies and accounting Decision making. To this extent, linear programming has proved useful in modeling diverse types of problems in planning, routing, scheduling assignment and design. David Charles H. [27], Nearing E. D., and Tucker A. W. [65], Koopmans, T.C. and Retire, S.[53], noted operational research is a mathematical method developed to solve problems related to tactical strategic operations. In the word of Lucy T.[60], allocation problems are concerned with the utilization of limited resources to be best advantage and linear programming is one of those techniques used in tackling this. In their own contribution, Hillier and Lieberman [40] stated that the development of linear programming was ranked among the most important scientific advances of the mid -20th century due to its extraordinary impact since 1950. They further opined that linear programming is a standard tool that has saved millions of dollars for most companies and business in the various industrialized countries of the world.

There are diverse opinions on the applicability of the linear programming technique to different management decision-making process. These opinions developed over a long period of time following continuous improvement on the application of the technique in solving practical business problems. Most literature in economic development supports the view that linear programming is a practical tool of analysis in allocating scarce resources to their optimal use and is of vital importance to the economics of underdeveloped countries. Danzig, G. B. [24] developed a powerful tool known as simplex method to solve linear programming problems and suggested this approach for solving business and industrial problems. In an allocation problem, where there are a number of activities to be performed, alternatives ways of doing them and limited resources or facilities for performing each activity in the most effective way, the management is faced with the problem of how best to combine these activities and resources in an optimal manner so that the overall efficiency is maximized. According to Charnes, A., Cooper, W.W. and Henderson [17], known as optimization problem and is approached using mathematical programming. Dowing E. T. [29], advocates that the lagrangian method should be used for any optimization subject to a single inequality constraint, the graphic approach for optimization subject to only two constraints, and the linear programming model for optimization subject to many inequality constraint and Dwivedi, D. [32], posit that linear programming is of great use in making business decision because it helps in measuring complex economic relations and thereby, provides an optimum solution to the problem of resource allocation.

Applications of Mathematical Programming

The early applications of mathematical programming (MP) were concerned with military planning and coordination among various projects and the efficient utilization of scarce resources. During the last five decades MP techniques are applied successfully to almost every sphere of human activity, such as industries, agriculture, engineering and scientific research etc.

An important application of MP techniques is seen in various statistical problems. The need of using these techniques in optimization problems arising in statistics is excellently described by Prof. C.R. Rao in Arthanari and Dodge [4], which is reproduced here: "*All statistical procedures are, in the ultimate analysis, solutions to suitably formulated optimization problems "mathematical programming problems"*. Whether it is designing a specific experiment, or planning a large scale survey for collection of data, or choosing a stochastic model to characterize observed data, or drawing inference from available data, such as estimation, testing of hypothesis and decision making, one has to choose an objective function and minimize or maximize it subject to given constraints or unknown parameters and inputs such as the cost involved. The classical optimization methods based on differential calculus are too restrictive, and are either inapplicable or difficult to apply in many situations that arise in statistical work. This, together with the lack of suitable numerical algorithms for solving optimizing equations, has placed severe limitations on the choice of objective functions and constraints and led to the development and use of some inefficient statistical procedures. Attempts have therefore been made during the last three decades to find other optimization techniques that have wider applicability and can be easily implemented with the available computing power.

One such technique that has the potential for increasing the scope for application of efficient statistical methodology is MP.

The fundamental paper by Charnes et al.[14] introduced the application of MP technique to statistics. In linear regression they choose an approach to minimize the sum the absolute deviations (MINAD) which is an alternative to the least square approach and formulated the MINAD problem as an LPP. An efficient modification of the simplex method introduced by Barrodate and Roberts [6] increased the possibility of using MINAD regression as an alternative to classical regression.

Some other applications of MP techniques to problems arising in statistical analysis are found in Cluster analysis, construction of BIBD and other designs, Reliability and Quality control.

Mathematical programming models and techniques are so widely used to a variety of disciplines emerging from almost every branch of science, industry, agriculture, engineering, management, planning, social and economic problems, medical science, business, military, statistical analysis etc., that it is difficult to provide a complete list of all applications of MP techniques.

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